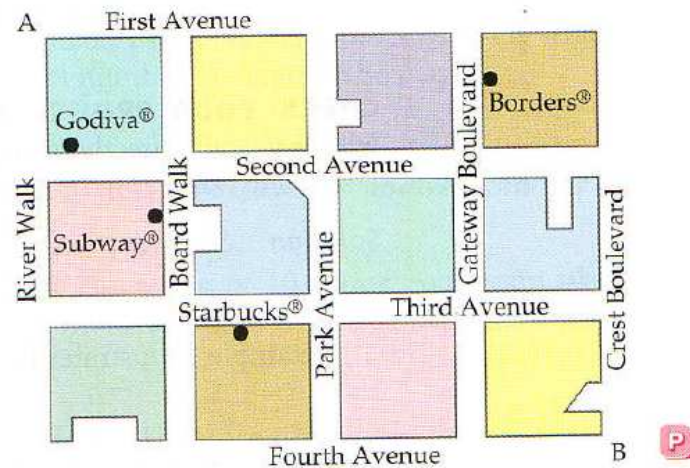


Section Européenne DNL mathématiques  
**Problem solving with Polya's strategy :**  
**Number of direct routes**

Source : *Mathematical excursions* by Aufman, Lockwood, Nation and Clegg.

Consider the map shown below. Allison wishes to walk along the streets from point A to point B. How many direct routes (no backtracking) can Allison take?



## Section Européenne DNL mathématiques

### Polya's problem solving strategy

Source : *Mathematical excursions* by Aufman, Lockwood, Nation and Clegg.

#### historical note



**George Polya**  
After a brief stay at Brown University, George Polya (pōl'yə) moved to Stanford University in 1942 and

taught there until his retirement. While at Stanford, he published 10 books and a number of articles for mathematics journals. Of the books Polya published, *How to Solve it* (1945) is one of his best known. In this book, Polya outlines a strategy for solving problems from virtually any discipline.

"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery." ■

#### Polya's Problem-Solving Strategy

Ancient mathematicians such as Euclid and Pappus were interested in solving mathematical problems, but they were also interested in *heuristics*, the study of the methods and rules of discovery and invention. In the seventeenth century, the mathematician and philosopher René Descartes (1596–1650) contributed to the field of heuristics. He tried to develop a universal problem-solving method. Although he did not achieve this goal, he did publish some of his ideas in *Rules for the Direction of the Mind* and his better-known work *Discourse de la Methode*.

Another mathematician and philosopher, Gottfried Wilhelm Leibnitz (1646–1716), planned to write a book on heuristics titled *Art of Invention*. Of the problem-solving process, Leibnitz wrote, "Nothing is more important than to see the sources of invention which are, in my opinion, more interesting than the inventions themselves."

One of the foremost recent mathematicians to make a study of problem solving was George Polya (1887–1985). He was born in Hungary and moved to the United States in 1940. The basic problem-solving strategy that Polya advocated consisted of the following four steps.

##### Polya's Four-Step Problem-Solving Strategy

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

Polya's four steps are deceptively simple. To become a good problem solver, it helps to examine each of these steps and determine what is involved.

**Understand the Problem** This part of Polya's four-step strategy is often overlooked. You must have a clear understanding of the problem. To help you focus on understanding the problem, consider the following questions.

- Can you restate the problem in your own words?
- Can you determine what is known about these types of problems?
- Is there missing information that, if known, would allow you to solve the problem?
- Is there extraneous information that is not needed to solve the problem?
- What is the goal?

**Devise a Plan** Successful problem solvers use a variety of techniques when they attempt to solve a problem. Here are some frequently-used procedures.

- Make a list of the known information.
- Make a list of information that is needed.
- Draw a diagram.
- Make an organized list that shows all the possibilities.
- Make a table or a chart.
- Work backwards.
- Try to solve a similar but simpler problem.
- Look for a pattern.
- Write an equation. If necessary, define what each variable represents.
- Perform an experiment.
- Guess at a solution and then check your result.
- Use indirect reasoning.

**Carry Out the Plan** Once you have devised a plan, you must carry it out.

- Work carefully.
- Keep an accurate and neat record of all your attempts.
- Realize that some of your initial plans will not work and that you may have to devise another plan or modify your existing plan.

**Review the Solution** Once you have found a solution, check the solution.

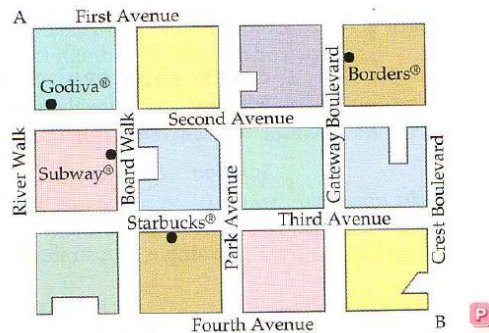
- Ensure that the solution is consistent with the facts of the problem.
- Interpret the solution in the context of the problem.
- Ask yourself whether there are generalizations of the solution that could apply to other problems.



**Section Européenne DNL mathématiques**  
**Apply Polya's strategy:**  
**Solve a similar but simpler problem**  
**Number of direct routes**

Source : *Mathematical excursions* by Aufman, Lockwood, Nation and Clegg.

Consider the map shown below. Allison wishes to walk along the streets from point A to point B. How many direct routes can Allison take?

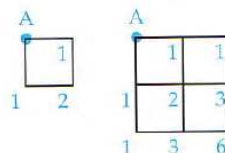


**Solution**

**Understand the Problem** We would not be able to answer the question if Allison retraced her path or traveled away from point B. Thus we assume that on a direct route, she always travels along a street in a direction that gets her closer to point B.

**Devise a Plan** The map in Figure 1.2 has many extraneous details. Thus we make a diagram that allows us to concentrate on the essential information. See the figure at the left.

Because there are many routes, we consider the similar but simpler diagrams shown below. The number at each street intersection represents the number of routes from point A to that particular intersection.



Look for patterns. It appears that the number of routes to an intersection is the *sum* of the number of routes to the adjacent intersection to its left and the number of routes to the intersection directly above. For instance, the number of routes to the intersection labeled 6 is the sum of the number of routes to the intersection to its left, which is three, and the number of routes to the intersection directly above, which is also three.

**Carry Out the Plan** Using the pattern discovered on the previous page, we see from the figure at the left that the number of routes from point A to point B is  $20 + 15 = 35$ .

**Review the Solution** Ask yourself whether a result of 35 seems reasonable. If you were required to draw each route, could you devise a scheme that would enable you to draw each route without missing a route or duplicating a route?

**CHECK YOUR PROGRESS 1** Consider the street map in Figure 1.2. Allison wishes to walk directly from point A to point B. How many different routes can she take if she wants to go past Starbucks on Third Avenue?

**Section Européenne DNL mathématiques**  
**Apply Polya's strategy:**  
**Solve a similar but simpler problem**  
**Number of direct routes**

Classe : Seconde

Objectifs :

- Leur donner un problème où *tout le monde* peut démarrer (on peut expérimenter)
- Découvrir une stratégie de résolution de problème : Schématiser la situation, expérimenter, démarrer par un cas simple que l'on généralise : L'art de ne pas rester paralysé devant un problème.
- Appliquer la méthode de Polya de résolution de problème.
- Assimilation de la méthode par une démarche active : « Check your progress » permet de réinvestir immédiatement.

Déroulement prévu pour la séance :

- Leur donner le problème et les laisser chercher seuls.
- Après quelques minutes, expliquez la question si besoin est.
- Leur distribuer le document « Polya's method » et de nouveau, les laisser chercher.
- (On peut éventuellement les mettre en groupe et demander un rapporteur par groupe ?)
- Leur distribuer le corrigé et leur demander de faire le « Check your progress »

Réaction des élèves : Pas encore testé ! Chers collègues, si vous le faites, racontez-moi.

Chers collègues : Pour me faire parvenir vos commentaires sur ce document et/ou échanger des idées sur la DNL, vous pouvez me contacter à [lhelmeg@yahoo.com](mailto:lhelmeg@yahoo.com).